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PROPERTIES OF A RANDOM CREEP PROCESS

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Abstract—The scatter in creep data is considered to be an inherent material property due to the manufacturing process. The material behaviour under steady state creep is modeled as a three-dimensional random process.

The statistical properties of the process are discussed. The probability density of the level of the maxima is derived.

The statistical properties of the structural deformation rate for a simple model problem, viz. pure bending of a beam lamina, are derived.

1. INTRODUCTION

The standard creep test refers to a specimen under constant load and at constant temperature. The scatter in deformation rate and rupture time between different specimens is usually large. The origin of the scatter may be global factors, such as random variation of load or temperature. Hayhurst[1] has shown that the scatter can be reduced, but not eliminated, through a rigorous control of the test situation. Local factors, leading to inhomogeneous creep, may be random temperature along the specimen or random material properties.

Random material properties was already suggested by Hoff[2]. Assuming Norton's creep law $\dot{\epsilon} = B\sigma^n$ to be valid, then the scatter may be attributed to random variations of B and n. Comparison with creep tests may show if the variation of any of the parameters tends to dominate. Cozzarelli and Huang[3, 4] considered n as a random process, due to a random distribution of impurities, and B as another uncoupled random process, due to a random temperature distribution. Some simple hyperstatic structures under constant load were analysed. It was found that the deformation rate shows large random fluctuation. Broberg and Westlund[5, 6] considered B as a random process due to the manufacturing process. Also here the influence of the random material parameter on the stresses and deformation rates of some simple structures was analysed. A volume effect was shown to exist, such that the variance of the deformation rate decreases with increased structural redundancy and material volume.

Extreme values of random processes was first analysed by Rice[7]. He modeled an electric noise current as a one-dimensional normal random process. The probability distribution of the level of the maxima was derived. The creep rupture of specimens with random material properties were analysed by Broberg and Westlund[8]. The rupture time was determined in statistical terms. A volume effect was shown to exist such that the probability distribution of the rupture time is strongly dependent on the material volume. Longuet-Higgins[9] modeled the height of sea-waves as a two-dimensional normal random process. The probability distribution of the maxima was derived. Nayak[10] modeled the height of a rough surface as a two-dimensional normal random process. The probability density of the level of the maxima and other statistical properties were derived.

Starting with the statistical properties of creep tests and the work of Broberg and Westlund[5, 6, 8] and Nayak[10] the material properties under steady state creep will now be modeled as a three-dimensional random process. The probability density of the level of the maxima will be determined. As a simple model problem a beam lamina under constant bending will be analysed. The curvature rate of a random cross-section will be determined in statistical terms.

2. THE RANDOM MATERIAL MODEL

2.1 *Statistical properties of the random process*

The material properties in the metal cast may be considered random and isotropic. During the metal forming process, such as rolling or extrusion, the material properties may become anisotropic. Chang and Grant[11] observed different strain rates between the grains of a coarse-grained aluminium specimen. The material properties on the microscopical scale cannot be considered. A macroscopical representation, compatible with the continuum theory, must be assigned. It is now assumed that the material properties are described by a stationary three-dimensional random process.

Observations from a number of creep tests, see [12], indicates that the scatter in steady state deformation rate is more or less independent of the applied load. This implies, assuming Norton's creep law $\dot{\epsilon} = B\sigma^n$ to be valid, that the scatter is due to random variations in B rather than in n. It is now assumed that the creep behaviour is described by Norton's creep law and that the random behaviour is described by variations in B only.

The principal steady state creep rates are given by

$$\dot{\epsilon}_j(\mathbf{x}) = \frac{3}{2}\dot{\epsilon}_0\left[\frac{\sigma_e(\mathbf{x})}{\sigma_n}\right]^{n-1}\frac{s_j(\mathbf{x})}{\sigma_n}C(\mathbf{x}) \ (j = 1, 2, 3) \tag{2.1}$$

where σ_e and s_j are the Mises effective stress and the principal stress deviators at $\mathbf{x} = (x_1, x_2, x_3)$. Moreover $\dot{\epsilon}_0$ and n are material constants and σ_n is a constant introduced for dimensional purposes. The random material behaviour is described by the stationary random process $C(\mathbf{x})$.

Walles[13] gave the scatter between different specimens a thorough statistical treatment. The steady state creep rates was shown to be close to log-normal distributed. It is now assumed that the principal steady state creep rates are log-normal distributed. The distribution function of C is

$$f(\eta) = \frac{1}{\eta s(2\pi)^{1/2}}\exp[-(\ln \eta + s^2/2)^2/2s^2] \tag{2.2}$$

with the expected value and variance

$$E[C] = 1 \qquad V[C] = e^{s^2} - 1. \tag{2.3}$$

If the material behaviour shall be compatible with the ordinary small strain theory, then the scatter must be small. When the variance of C tends to zero, then the log-normal distribution tends to normal. The random process is conveniently reformulated as

$$C(\mathbf{x}) = 1 + \alpha H(\mathbf{x}) \tag{2.4}$$

with $|\alpha H| \ll 1$. The distribution function of αH is

$$f(\eta) = (2\pi\alpha)^{-1/2}\exp(-\eta^2/2\alpha) \tag{2.5}$$

with the expected value and variance

$$E[\alpha H] = 0 \qquad V[\alpha H] = \alpha. \tag{2.6}$$

For a complete knowledge of the random process the auto-correlation, or its Fourier transform, must be known. The spectral density may, on physical grounds, be described as a band-pass white noise. It is now assumed to be low-pass white noise, in the general anisotropic case given by

$$\Phi(\boldsymbol{\omega}) = \Phi_0 = \frac{3\alpha}{4\pi\beta_1\beta_2\beta_3}u\left[1 - \left(\frac{\omega_1}{\beta_1}\right)^2 - \left(\frac{\omega_2}{\beta_2}\right)^2 - \left(\frac{\omega_3}{\beta_3}\right)^2\right] \tag{2.7}$$

with

$$\beta_1 = \beta \qquad \beta_2 = k_{m2}\beta \qquad \beta_3 = k_{m3}\beta. \tag{2.8}$$

Here β is a material constant and k_{m1} and k_{m2} are coefficients of material anisotropy, and u denotes the unit step function. The auto-correlation is given by

$$R(\mathbf{x}_0) = E[\alpha H(\mathbf{x})\alpha H(\mathbf{x}+\mathbf{x}_0)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\omega_1\,\mathrm{d}\omega_2\,\mathrm{d}\omega_3 \Phi(\boldsymbol{\omega})\,\mathrm{e}^{i\mathbf{x}\cdot\boldsymbol{\omega}}. \tag{2.9}$$

The variable substitutions

$$y_j = x_{0j}\beta_j \qquad \xi_j = \omega_j/\beta_j \qquad (j = 1, 2, 3) \tag{2.10}$$

yield

$$R(\mathbf{y}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\xi_1\,\mathrm{d}\xi_2\,\mathrm{d}\xi_3\Phi(\boldsymbol{\xi})\,\mathrm{e}^{i\mathbf{y}\cdot\boldsymbol{\xi}} \tag{2.11}$$

and

$$\Phi(\boldsymbol{\xi}) = \frac{3\alpha}{4\pi}u(1-\xi_1^2-\xi_2^2-\xi_3^2). \tag{2.12}$$

The solution of the radially symmetric integral is given by Sneddon [14]

$$R(\mathbf{x}_0) = 3\left(\frac{\pi}{2}\right)^{1/2}\alpha\frac{J_{3/2}(r_0)}{r_0^{3/2}} = \alpha\sum_{j=0}^{\infty}\frac{(-r_0^2/4)^j\Gamma(5/2)}{j!\Gamma(j+5/2)} \tag{2.13}$$

with

$$r_0 = [(\beta x_{10})^2 + (k_{m2}\beta x_{20})^2 + (k_{m3}\beta x_{30})^2]^{1/2}. \tag{2.14}$$

Here $J_{3/2}$ denotes the 3/2-order Bessel function and Γ denotes the gamma function.

The statistical properties of the process may be deduced from the moments of the spectral density

$$m_{pqr} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\omega_1\,\mathrm{d}\omega_2\,\mathrm{d}\omega_3\omega_1^p\omega_2^q\omega_3^r\Phi(\boldsymbol{\omega}). \tag{2.15}$$

Insertion of eqn (2.7) yields non-zero moments only for even p, q and r

$$m_{pqr} = \frac{3\alpha\beta^{p+q+r}}{4\pi}k_{2m}^q k_{3m}^r\frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)\Gamma\left(\frac{r+1}{2}\right)}{\Gamma\left(\frac{p+q+r+5}{2}\right)}. \tag{2.16}$$

2.2 *Extreme values of the random process*

An extreme value analysis is most conveniently carried out on the normal process

$$L(\mathbf{x}) = \ln C(\mathbf{x}) \tag{2.17}$$

or in the ordinary small scatter formulation

$$L(\mathbf{x}) = \alpha H(\mathbf{x}). \tag{2.18}$$

A stationary point will be an extremum point if the quadratic form (summation rule)

$$Q = \Lambda_{ij}x_ix_j \tag{2.19}$$

is either positive or negative definite. Here Λ denotes the matrix of second derivatives of L. The condition for a maximum is that the invariants of the matrix

$$\left.\begin{aligned} I_1 &= \Lambda_{ii} < 0 \\ I_2 &= [(\Lambda_{ii})^2 - \Lambda_{ij}\Lambda_{ij}]/2 > 0 \\ I_3 &= |\Lambda| < 0. \end{aligned}\right\} \tag{2.20}$$

It is convenient to introduce, see [10],

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10} = L''_{11}, L''_{12}, L''_{22}, L''_{23}, L''_{33}, L''_{31}, L, L'_1, L'_2, L'_3. \tag{2.21}$$

The expected number of maxima of level $\lambda_7 = \lambda$, in the volume $\mathrm{d}V$, is given by

$$E[M(\lambda)]\,\mathrm{d}V = \int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_R \mathrm{d}\lambda_1\,\mathrm{d}\lambda_2\,\mathrm{d}\lambda_3\,\mathrm{d}\lambda_4\,\mathrm{d}\lambda_5\,\mathrm{d}\lambda_6 \int\!\!\int\!\!\int_{\Delta V} \mathrm{d}\lambda_8\,\mathrm{d}\lambda_9\,\mathrm{d}\lambda_{10}\,p(\boldsymbol{\lambda}). \tag{2.22}$$

Here p denotes the joint probability density. The region of integration R is given by eqn (2.20). The increments of the first derivatives in $\mathrm{d}V$ are

$$\mathrm{d}\lambda_8\,\mathrm{d}\lambda_9\,\mathrm{d}\lambda_{10} = \left|\frac{\partial(\lambda_8, \lambda_9, \lambda_{10})}{\partial(x_1, x_2, x_3)}\right| \mathrm{d}V = |\Lambda|\,\mathrm{d}V. \tag{2.23}$$

According to the central limit theorem the joint probability is

$$p(\boldsymbol{\lambda}) = (2\pi)^{-5}|S|^{-1/2}\exp(-S^{-1}_{ij}\lambda_i\lambda_j/2) \tag{2.24}$$

where the elements of the matrix of covariances are given by

$$S_{ij} = E[\lambda_i\lambda_j]. \tag{2.25}$$

The matrix may be expressed in the moments of the spectral density

$$S = \begin{bmatrix} m_{400} & m_{310} & m_{220} & m_{211} & m_{202} & m_{301} & -m_{200} & 0 & 0 & 0 \\ & m_{220} & m_{130} & m_{121} & m_{112} & m_{211} & -m_{110} & 0 & 0 & 0 \\ & & m_{040} & m_{031} & m_{022} & m_{121} & -m_{020} & 0 & 0 & 0 \\ & & & m_{022} & m_{013} & m_{112} & -m_{011} & 0 & 0 & 0 \\ & \text{SYMMETRIC} & & & m_{004} & m_{103} & -m_{002} & 0 & 0 & 0 \\ & & & & & m_{202} & -m_{101} & 0 & 0 & 0 \\ & & & & & & m_{000} & 0 & 0 & 0 \\ & & & & & & & m_{200} & m_{110} & m_{101} \\ & & & & & & & & m_{020} & m_{011} \\ & & & & & & & & & m_{002} \end{bmatrix}. \tag{2.26}$$

The expected number of all maxima is given by

$$E[S(\infty)] = (2\pi)^{-5}|S|^{-1/2}\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_R \mathrm{d}\lambda_1\,\mathrm{d}\lambda_2\,\mathrm{d}\lambda_3\,\mathrm{d}\lambda_4\,\mathrm{d}\lambda_5\,\mathrm{d}\lambda_6 \int_{-\infty}^{\infty} \mathrm{d}\lambda_7$$
$$\times |\Lambda|\exp(-S^{-1}_{ij}\lambda_i\lambda_j/2). \tag{2.27}$$

The probability density of maxima of level λ is finally calculated as

$$p_m(\lambda) = \frac{E[M(\lambda)]}{E[S(\infty)]}. \tag{2.28}$$

Insertion of the moments of the spectral density eqn (2.16) and the new variables

$$\begin{aligned}
&l_1 = \alpha^{-1/2}\beta^{-2}\lambda_1 \quad l_2 = \alpha^{-1/2}k_{m2}^{-1}\beta^{-2}\lambda_2 \quad l_3 = \alpha^{-1/2}k_{m2}^{-2}\beta^{-2}\lambda_3\\
&l_4 = \alpha^{-1/2}k_{m2}^{-1}k_{m3}^{-1}\beta^{-2}\lambda_4 \quad l_5 = \alpha^{-1/2}k_{m3}^{-2}\beta^{-2}\lambda_5\\
&l_6 = \alpha^{-1/2}k_{m3}^{-1}\beta^{-2}\lambda_6 \quad l_7 = \alpha^{-1/2}\lambda_7 \quad l_8 = \alpha^{-1/2}\beta^{-1}\lambda_8\\
&l_9 = \alpha^{-1/2}k_{m2}^{-1}\beta^{-1}\lambda_9 \quad l_{10} = \alpha^{-1/2}k_{m3}^{-1}\beta^{-1}\lambda_{10}
\end{aligned} \tag{2.29}$$

followed by matrix inversion gives the expected number of maxima of level $l_7 = l$

$$\begin{aligned}
E[M(l)] = \frac{1}{196}\left(\frac{35}{2\pi}\right)^5 \alpha^{-1/2}k_{m2}k_{m3}\beta^3 \int_{-\infty}^{0} \mathrm{d}l_1 \int_{-\infty}^{(l_1l_3)^{1/2}} \mathrm{d}l_2 \int_{-\infty}^{0} \mathrm{d}l_3 \int_{-\infty}^{(l_3l_5)^{1/2}} \mathrm{d}l_4 \int_{-\infty}^{0} \mathrm{d}l_5 \int_{-\infty}^{(l_1l_5)^{1/2}} \mathrm{d}l_6\\
\times |l_1l_3l_5 - l_1l_4^2 - l_3l_6^2 - l_5l_2^2 - 2l_2l_4l_6| \exp\Big[-\tfrac{5}{8}(21l_1^2 + 14l_1l_3\\
+ 14l_1l_5 + 14l_1l + 28l_2^2 + 21l_3^2 + 14l_3l_5 + 14l_3l + 28l_4^2 + 21l_5^2\\
+ 14l_5l + 28l_6^2 + 5l^2)\Big].
\end{aligned} \tag{2.30}$$

Equation (2.30) is most conveniently solved by numerical integration. The probability density of the maxima of level λ, see eqn (2.28), is independent of the material anisotropy.

In order to achieve an analytical solution a simple model problem will now be considered.

3. APPLICATION TO A MODEL PROBLEM RESTRICTED TO TWO DIMENSIONS

3.1 *The model problem*

A beam lamina under constant bending will be analysed. Only the properties of a random cross-section x_3 will be considered, see Fig. 1. The index x_3 is hereafter dropped. The analysis may well be extended to a beam of finite length, as considered by Huang and Cozzarelli[4] in the limited case of random material parameters along the neutral axis only.

Equilibrium of the beam lamina requires

$$M = \int_{-B_1}^{B_1} \mathrm{d}x_1 \int_{-B_2}^{B_2} \mathrm{d}x_2 x_2\sigma(\mathbf{x}). \tag{3.1}$$

The width and height of the cross-section are expressed as

$$B_1 = B \quad B_2 = k_gB \tag{3.2}$$

where k_g is a coefficient of geometry.

For small geometrical changes the strain of the lamina is

$$\epsilon(\mathbf{x}) = \kappa x_2 \tag{3.3}$$

where κ is the curvature of the center-plane.

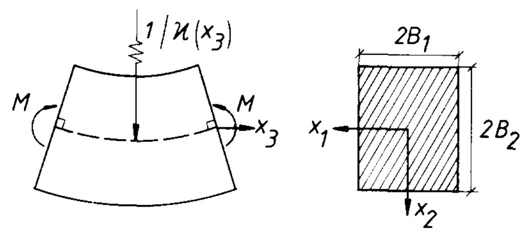

Fig. 1. Model problem.

The constitutive eqn (2.1) for the uni-axial stress state is

$$\dot{\epsilon}(\mathbf{x}) = \dot{\epsilon}_0 \left[\frac{\sigma(\mathbf{x})}{\sigma_n}\right]^n [1 + \alpha H(\mathbf{x})]. \tag{3.4}$$

In order to take into account also compressive stresses, n is restricted to be an odd integer.

The solution will be carried out in the new variables

$$\left.\begin{array}{lll} \eta_1 = x_1/B & \eta_2 = x_2/k_g B & m = M/k_g^2 B^3 \sigma_n \\ s = \sigma/\sigma_n & \dot{h} = k_g B \dot{\kappa}/\dot{\epsilon}_0. & \end{array}\right\} \tag{3.5}$$

From eqns (3.3) and (3.4) follows

$$s(\boldsymbol{\eta}) = \dot{h}^{1/n} \eta_2^{1/n} [1 + \alpha H(\boldsymbol{\eta})]^{-1/n}. \tag{3.6}$$

Insertion in eqn (3.1) gives the curvature rate

$$\dot{h} = m^n \left\{ \int_{-1}^{1} \mathrm{d}\eta_1 \int_{-1}^{1} \mathrm{d}\eta_2 \eta_2^{1+1/n} [1 + \alpha H(\boldsymbol{\eta})]^{-1/n} \right\}^{-n} \tag{3.7}$$

A series development of the order $0(\alpha^3)$ gives

$$\dot{h} = m^n I_0^{-n} \left[1 + \frac{I_1}{I_0} - \frac{n+1}{2n} \frac{I_2}{I_0} + \frac{n+1}{2n} \left(\frac{I_1}{I_0}\right)^2 \right] + 0(\alpha^3) \tag{3.8}$$

where

$$I_j = \int_{-1}^{1} \mathrm{d}\eta_1 \int_{-1}^{1} \mathrm{d}\eta_2 \eta_2^{1+1/n} [\alpha H(\boldsymbol{\eta})]^j. \tag{3.9}$$

3.2 *The statistical analysis*

The expected value and variance of the curvature rate may be deduced from the corresponding properties of the integrals I_j of eqn (3.9). Insertion of eqn (2.6) and (2.9) gives

$$E[I_1] = \int_{-1}^{1} \mathrm{d}\eta_1 \int_{-1}^{1} \mathrm{d}\eta_2 \eta_2^{1+1/n} E[\alpha H(\boldsymbol{\eta})] = 0 \tag{3.10}$$

$$E[I_2] = \int_{-1}^{1} \mathrm{d}\eta_1 \int_{-1}^{1} \mathrm{d}\eta_2 \eta_2^{1+1/n} E[\alpha^2 H^2(\boldsymbol{\eta})] = \alpha I_0 \tag{3.11}$$

$$E[I_1^2] = \int_{-1}^{1} \mathrm{d}\eta_1 \int_{-1}^{1} \mathrm{d}\eta_2 \int_{-1}^{1} \mathrm{d}\zeta_1 \int_{-1}^{1} \mathrm{d}\zeta_2 (\eta_2 \zeta_2)^{1+1/n} E[\alpha H(\boldsymbol{\eta}) \alpha H(\boldsymbol{\xi})] = \alpha I_0^2 W_n(b, k) \tag{3.12}$$

Here W_n is a structural geometry factor, deduced by insertion of eqn (2.13) with $x_{30} = 0$, thus

$$W_n(b, k) = 3\left(\frac{\pi}{2}\right)^{1/2} \frac{(2n+1)^2}{16n^2} \int_{-1}^{1} \mathrm{d}\eta_1 \int_{-1}^{1} \mathrm{d}\eta_2 \int_{-1}^{1} \mathrm{d}\zeta_1 \int_{-1}^{1} \mathrm{d}\zeta_2 \int_{-1}^{1} \mathrm{d}\zeta_2 (\eta_2 \zeta_2)^{1+1/n} \frac{J_{3/2}(r_0)}{r_0^{3/2}} \tag{3.13}$$

where

$$r_0 = b[(\eta_1 - \zeta_1)^2 + k^2(\eta_2 - \zeta_2)^2]^{1/2} \tag{3.14}$$

and

$$b = \beta B \qquad k = k_g k_{m2}. \tag{3.15}$$

The dependence of W_n upon n can be shown to be very small. A good approximation for all values of n, with an error less than 5%, is

$$W_n = W_1 \equiv W. \tag{3.16}$$

Insertion of the variable transformations

$$t_1 = \eta_1 - \zeta_1, \quad t_2 = \eta_2 - \zeta_2, \quad s_1 = \eta_1 + \zeta_1, \quad s_2 = \eta_2 + \zeta_2 \tag{3.17}$$

and the series expansion of $J_{3/2}$ eqn (2.13) followed by partial integration on s_1 and s_2 yield

$$W = \frac{3}{40}\int_0^2 dt_1 \int_0^2 dt_2 (2-t_1)(12-30t_2+20t_2^2-t_2^5) \times \sum_{j=0}^{\infty} \frac{(-1/4)^j \Gamma(5/2)}{j!\Gamma(j+5/2)} b^{2j} \times \sum_{l=0}^{j} \binom{j}{l} t_1^{2(j-l)} k^{2l} t_2^{2l}. \tag{3.18}$$

Reversion of the order of integration and summation gives

$$W = 72 \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(5/2)}{j!\Gamma(j+5/2)} b^{2j} \sum_{l=0}^{j} \binom{j}{l} k^{2l} \times \frac{2l^2+l+1}{[2(j-l)+2][2(j-l)+1](2l+1)(2l+2)(2l+3)(2l+6)} \tag{3.19}$$

which is calculated for two values of k, see Fig. 2.

Finally the expected value and variance of the curvature rate follows as

$$E[\dot{h}] = m^n \left(\frac{4n}{2n+1}\right)^{-n} \left[1 - \frac{n+1}{2n}\alpha(1-W)\right] + 0(\alpha^2) \tag{3.20}$$

$$V[\dot{h}] = m^{2n} \left(\frac{4n}{2n+1}\right)^{-2n} \alpha W + 0(\alpha^2). \tag{3.21}$$

The scatter of the structural deformation rate is seen to be strongly dependent on the material volume and the structural redundancy.

3.3 *The probability density of maxima*

The extreme value density of a radom x_3-surface follows from part 2.2 above by making all derivatives on x_3 equal to zero. The expected number of maxima of level $\lambda_7 = \lambda$, per unit area, is given by

$$E[M(\lambda)] = (2\pi)^{-3} |S|^{-1/2} \int_{-\infty}^{0} d\lambda_1 \int_{-\infty}^{0} d\lambda_3 \int_{-\infty}^{(\lambda_1\lambda_3)^{1/2}} d\lambda_2 (\lambda_1\lambda_3 - \lambda_2^2) \exp(-S_{ij}^{-1}\lambda_i\lambda_j/2) \tag{3.22}$$

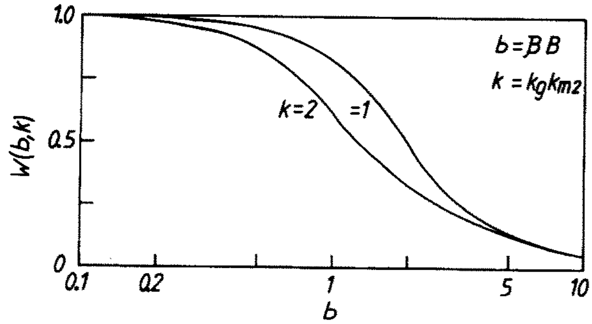

Fig. 2. Structural geometry factor.

where

$$S = \begin{bmatrix} m_{40} & 0 & m_{22} & -m_{20} & 0 & 0 \\ & m_{22} & 0 & 0 & 0 & 0 \\ & \text{SYMMETRIC} & m_{04} & -m_{02} & 0 & 0 \\ & & & m_{00} & 0 & 0 \\ & & & & m_{20} & 0 \\ & & & & & m_{02} \end{bmatrix}. \quad (3.23)$$

Insertion of the moments of the spectral density eqn (2.16) with $r = 0$ and the new variables

$$\left.\begin{aligned} l_{10} &= \alpha^{-1/2}\beta^{-2}\lambda_1 \quad & l_2 &= \alpha^{-1/2}k_{m2}^{-1}\beta^{-2}\lambda_2 \\ l_{30} &= \alpha^{-1/2}k_{m2}^{-2}\beta^{-2}\lambda_3 \quad & l_7 &= \alpha^{-1/2}\lambda_7 \end{aligned}\right\} \quad (3.24)$$

followed by matrix inversion yield

$$E[M(l)] = c_0\int_{-\infty}^{0} dl_{10}\int_{-\infty}^{0} dl_{30}\int_{-\infty}^{(l_{10}l_{30})^{1/2}} dl_2(l_{10}l_{30} - l_2^2)$$
$$\times \exp\left[-\frac{5}{6}(14l_{10}^2 + 7l_{10}l_{30} + 7l_{10}l + 21l_2^2 + 14l_{30}^2 + 7l_{30}l + 2l^2)\right] \quad (3.25)$$

where

$$c_0 = \frac{1}{7}\left(\frac{35}{2\pi}\right)^3 (84)^{-1/2}\alpha^{-1/2}k_{m2}\beta^2. \quad (3.26)$$

The linear transformation of variables

$$l_1 = (l_{10} + l_{30})/2 \qquad l_3 = (l_{10} - l_{30})/2 \quad (3.27)$$

gives

$$E[M(l)] = 2c_0\int_{-\infty}^{0} dl_1\int_{l_1}^{-l_1} dl_3\int_{-\infty}^{(l_1^2-l_3^2)^{1/2}} dl_2(l_1^2 - l_2^2 - l_3^2)$$
$$\times \exp\left[-\frac{5}{6}(35l_1^2 + 14l_1l + 21l_2^2 + 21l_3^2 + 2l^2)\right]. \quad (3.28)$$

Partial integration of eqn (3.28) gives the expected number of maxima of level l, erf denotes the error function,

$$E[M(l)] = \frac{c_1}{20\pi}\{6(7)^{1/2}l\, e^{-5l^2/3}\} + 7(6\pi)^{1/2}(l^2 - 1)\, e^{-l^2/2}$$
$$\left[1 + \operatorname{erf}\left(\frac{7}{6}\right)^{1/2} l\right] + 5(15\pi)^{1/2}\, e^{-15l^2/16}\left[1 + \operatorname{erf}\left(\frac{35}{48}\right)^{1/2} l\right] \quad (3.29)$$

and the expected number of all maxima

$$E[S(\infty)] = c_1 \quad (3.30)$$

where

$$c_1 = (7\pi)^{-1}12^{-1/2}\alpha^{-1/2}k_{m2}\beta^2. \quad (3.31)$$

The probability density of the maxima of level l follows from eqn (2.28), it is drawn in Fig. 3.

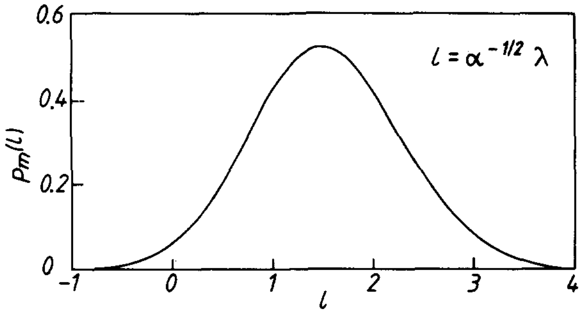

Fig. 3. Probability density of maxima of level L.

4. DISCUSSION

The material behaviour under steady state creep has been modeled as a stationary three-dimensional random process. The randomness in Norton's creep law has been described as a variation in one parameter only. A more sophisticated model of scatter should take into consideration a variation in both parameters. The random material behaviour may also be coupled to a more realistic constitutive equation.

The random process has been assumed to be log-normal distributed. This is consistent with a normal distributed random temperature with small variations. The structural deformation rates, deduced by integration of the log-normal local material parameter, will not be log-normal distributed. The small scatter case of the normal distributed local material parameter will be consistent with normal distributed structural deformations.

The extreme value analysis has been coupled to a definite spectral density, viz. anisotropic low-pass white noise, in order to simplify the analysis. The probability density of the level of the maxima is seen to be independent of the anisotropy.

In order to achieve analytical solutions a model problem, restricted to two dimensions, has been considered. A volume effect has been shown to exist, such that the influence of local variation of the material parameter on the structural behaviour decreases with increased material volume and structural redundancy.

Experiments sufficient to determine how large part of the creep data scatter that really is due to random material properties has yet not been conducted.

REFERENCES

1. D. R. Hayhurst, The effects of test variables on scatter in high temperature tensile creep-rupture data. *Int. J. Mech. Sci.* **16**, 829 (1974).
2. N. J. Hoff, Necking and rupture of rods subjected to constant tensile loads. *J. Appl. Mech.* **20**, 105 (1953).
3. F. A. Cozzarelli and W. N. Huang, Effect of random material parameters on nonlinear steady creep solutions. *Int. J. Solids Structures* **7**, 1477 (1971).
4. W. N. Huang and F. A. Cozzarelli, Steady creep bending in a beam with random material parameters. *J. Franklin Instit.* **294**, 323 (1972).
5. H. Broberg and R. Westlund, Creep scatter as an inherent material property. *Archiv. Mech.* **31**, 165 (1979).
6. H. Broberg and R. Westlund, Creep in structures with random material properties. *Int. J. Solids Structures* **14**, 365 (1978).
7. S. O. Rice, Mathematical analysis of random noise. *Bell System Tech. J.* **23** (1944); **24**, 46 (1945). Reprinted in : *Selected Papers on Noise and Stochastic Processes* (Edited by N. Wax). Dover, New York (1954).
8. H. Broberg and R. Westlund, Creep rupture of specimens with random material properties. *Int. J. Solids Structures* **14**, 959 (1978).
9. M. S. Longuet-Higgins, The statistical analysis of a random moving surface. *Phil. Trans. R. Soc.* Series A, **249**, 321 (1957).
10. P. R. Nayak, Random process model of rough surfaces. *J. Lubrication Tech.* **93**, 398 (1971).
11. H. C. Chang and N. J. Grant, Inhomogeneity in creep deformation of coarse-grained high purity aluminum. *Trans. AIME* **197**, 1175 (1953).
12. H. Broberg, A probabilistic interpretation of creep rupture curves. *Archiv. Mech.* **25**, 871 (1973).
13. K. F. A. Walles, Random and systematic factors in the scatter of creep data. Aeronautical Research Council, C. P. No. 935, H.M.S.O., London (1967).
14. I. N. Sneddon, *The Use of Integral Transforms.* McGraw-Hill, New York (1972).